Journal of Engineering Physics and Thermophysics, Vol. 83, No. 3, 2010

IDENTIFICATION OF THE THERMOPHYSICAL CHARACTERISTICS OF MATERIALS

V. G. Zverev,[a] **V. A. Nazarenko,**[b] **and A. V. Teploukhov**[b]

UDC 536.24:510.5

From a unified methodological standpoint, the coefficient of the inverse problem of determining the thermophysical characteristics of a material from temperature measurements at its depth in the approximation of a semi-infinite body, single-layer and two-layer (with a layer of an ideal conductor) plate of finite thickness has been solved. The proposed technique does not employ operations of the smoothing of experimental data and allows one to remove the restrictions on the choice of the heating regime of the specimen of the material and to expand the range of measurements by the Fourier number.

Keywords: *thermophysical experiment, thermophysical characteristics, inverse heat conduction problem, semi-infinite body, single- and two-layer plates.*

Introduction. Heatproof composite materials are widely used in practice to ensure stable operation of modern technical facilities under extreme service conditions [1]. The negative influence of external factors on these materials is capable of degrading considerably their heatproof properties; therefore, of great value is the problem of monitoring the thermal characteristics of materials and articles in application to the coating–backing system. Knowledge of the thermophysical characteristics is also needed for carrying out calculations of thermal fields in structures and for solving the problems of the design and verification of the optimal parameters of heatproof systems. At the present time, theoretical models do not allow one to calculate, with sufficient accuracy, the thermophysical characteristics of structurally complex heatproof composite materials; therefore, a thermophysical experiment constitutes the main source of information on their properties [2, 3].

The governing parameters of the process of heat transfer in a solid body are the coefficient of thermal conductivity λ and the specific heat c of a material [4]. A combination of these parameters yields the coefficient of thermal diffusivity $a = \lambda/(\rho c)$ that represents the rate of the rearrangement of a heat field and plays an important part in the process of heat transfer. The heat capacity is the thermal characteristic of the equilibrium state of a material; therefore calorimetric methods are preferable for its direct determination. Knowledge of the heat capacity allows one to identify the heat conducting properties of the material $\lambda = a(\rho c)$ by investigating the coefficient a.

The theoretical basis for the methods of determining the thermophysical characteristics of materials is the differential equation of heat conduction theory to describe the thermal processes occurring in a solid body [4–6]. The complex structure of its analytical solution depends substantially on the specimen geometry, type of thermal effect, and on the stage of the process. Therefore in practical solution of coefficient inverse problems the carrying out of a thermal experiment that leads to explicit expressions for determining the thermophysical coefficients has become most popular. This imposes certain limitations on the choice of the regime of specimen heating and on the possible range of measurements by the Fourier number. The use of general analytical solutions makes it possible to remove these limitations and raise the accuracy of identification of thermophysical characteristics.

The aim of the present work was to develop the methodology and extend the possibilities of arranging a thermal experiment for determining the thermophysical coefficients (a, λ) by solving an inverse heat conduction problem [7, 8].

Thermophysical Experiment, Statement of a Direct and Inverse Heat Conduction Problems. It is most convenient to determine the thermophysical properties of materials on the basis of experimental investigation of the one-dimensional nonstationary thermal processes occurring in them. The simplest external effect that corresponds to the

[a]Tomsk State University, 36 Lenin Ave., Tomsk, 634050, Russia; email: zverev@niipmm.tsu.ru; [b]Moscow Heat Engineering Institute, 10 Berezovaya Ave., Moscow, 127273, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 3, pp. 614–621, May–June, 2010. Original article submitted July 17, 2009.

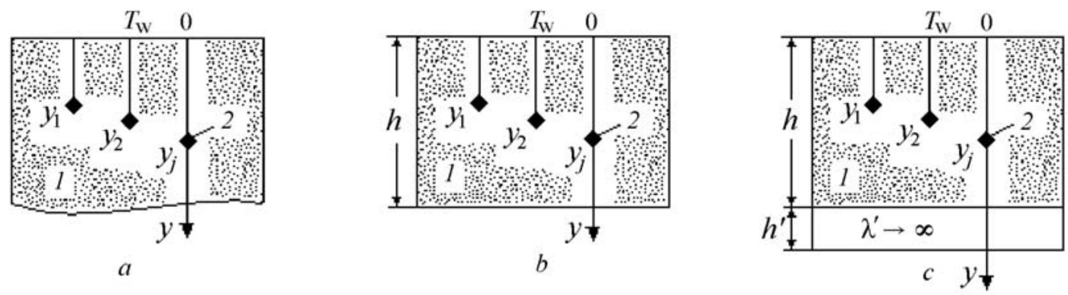


Fig. 1. Schematic of a thermophysical experiment (a, semi-infinite body; b, plate of thickness h; c, plate on metal backing of thickness h'): 1) test specimen; 2) sensor at the depth y_j.

boundary condition of the first kind is isothermal heating of the specimen surface. As a thermal source one can use, for this purpose, a massive metallic body brought into an ideal thermal contact with the working surface of the material being investigated.

Figure 1 shows the scheme of carrying out a thermophysical experiment for specimens in the form of a plane semi-infinite body (a), a homogeneous plate of thickness h (b), and a plate with a layer of an ideal conductor — a metal backing of thickness h' (c). Case (a) corresponds to the initial stage of the process and is widely used in practice, since it is convenient for carrying out an analysis and processing the results of measurements [2, 3]. However, a thick enough specimen is needed for its realization. In variant (b) it is possible to avoid the breakdown of the material structure by placing a temperature sensor on the heat-insulated back surface of the specimen. Case (c) is suitable for studying and monitoring the thermophysical characteristics of coatings on metal backings, also without any disturbance of the material structure.

Let the initial temperature of a specimen be $T(y, 0) = T_0 = 0$. At time $t = 0$ the surface temperature changes abruptly by the value $T(0, t) = T_c$ and further remains unchanged; the back side of the plate is assumed to be thermally insulated. The thermophysical properties of the material are constant. It should be noted that at the initial stage of heating the process develops in the zone adjoining the surface, and any specimen of finite thickness maintains the features of a semi-infinite medium. The mathematical description of the problem of heating has the form [4–6]

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial y^2}, \quad a = \frac{\lambda}{\rho c}, \quad t > 0; \tag{1}$$

$$t = 0: \quad T(y, 0) = T_0 = 0; \tag{2}$$

$$y = 0: \quad T(0, t) = T_w. \tag{3}$$

The boundary-value condition on the back surface of the specimen for the cases shown in Fig. 1a and b is written as

$$y = \infty, \quad y = h: \quad \frac{\partial T}{\partial y} = 0. \tag{4}$$

For the specimen on the metal backing (Fig. 1c), which is a source of thermocapacitive resistance, the boundary-value condition has the form [4–6]

$$y = h: \quad -\lambda\frac{\partial T}{\partial y} = \rho' c' h' \frac{\partial T}{\partial t}.$$

The determination of $T(y, t)$ by solving Eq. (1) with initial condition (2) and boundary-value conditions (3) and (4) at the known thermophysical characteristics represents a direct heat-conduction problem.

Let, as a results of carrying out a thermophysical experiment, the following values of temperature be obtained: $Y(y_j,\ t_{i,j})$, $1 \le j \le M$, $1 \le i \le N_j$. The determination of the coefficients a or λ from the known values of temperature $Y(y_j,\ t_{i,j})$ at time instants $t_{i,j}$ at the points of the specimen y_j lies in the essence of the coefficient inverse heat conduction problem [7, 8]. It is connected with the minimization of the functional of the summed squares of deviations of the experimental values $Y(y_j,\ t_{i,j})$ of temperature from those $\theta(y_j,\ t_{i,j})$ calculated over all selected time points and sensors [7, 8]:

$$J(a) = \sum_{j=1}^{M} \sum_{i=1}^{N_j} \left\{ Y(y_j, t_{i,j}) - \theta(y_j, t_{i,j}) \right\}^2 \to \min . \tag{5}$$

Analytical Solution of the Heat Conduction Problem. The analytical solution of problem (1)–(4) has the simplest form in the case of a semi-infinite body [4–6] (see Fig. 1a):

$$\theta(u) = \frac{T(y,t) - T_0}{T_\mathrm{w} - T_0} = 1 - \mathrm{erf}(u)\,, \quad u = \frac{y}{2\sqrt{at}}\,, \quad \mathrm{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-\xi^2)\, d\xi\,. \tag{6}$$

Here erf(u) is the integral of probabilities which was tabulated in many applications [9]. The applicability of this approximation to a plate of finite thickness h is restricted by the condition $4\sqrt{at} < h$, which corresponds to small values of the number $\mathrm{Fo} = (at)/h^2 < 0.0625$.

A more complex form of the exact solution of the boundary-value problem (1)–(4) in the form of an infinite series is encountered in the case of a plate of finite thickness (see Fig. 1b) [4–6]:

$$\theta(\eta, \mathrm{Fo}) = \frac{T(y,t) - T_0}{T_\mathrm{w} - T_0} = 1 - \sum_{n=1}^{\infty} A_n \cos[\mu_n(1-\eta)] \exp(-\mu_n^2 \mathrm{Fo})\,, \tag{7}$$

$$\mu_n = (2n-1)\frac{\pi}{2}\,; \quad A_n = (-1)^{n+1}\frac{2}{\mu_n}\,; \quad \mathrm{Fo} = \frac{at}{h^2}\,; \quad \eta = \frac{y}{h}\,.$$

Solution (7) depends on two variables $\theta(\eta,\ \mathrm{Fo})$; its nomograms are given in [4–6]. Owing to the complex structure, it is rarely used in practice. At large values of $\mathrm{Fo} > 0.4$ the series in (7) rapidly converges, and one can restrict oneself to its first term [4, 5]. This corresponds to the onset of a regular stage of the thermal process, which makes it possible to obtain the explicit connection between the thermal diffusivity coefficient a and the rate of superheating of an arbitrary inner layer. This approach underlies G. M. Kondratiev's method of a regular a calorimeter [10]. When $\mathrm{Fo} \to 0$, the convergence of the series in (7) is impaired, and an increase in the number of terms is required [4]. In this case, there exists another form of the representation of the solution for which it is sufficient to use several terms of the series [4]:

$$\theta(\eta, \mathrm{Fo}) = \frac{T(y,t) - T_0}{T_\mathrm{w} - T_0} = \sum_{n=1}^{\infty} (-1)^{n+1} \left\{ \mathrm{erfc}\left[\frac{(2n-1) - (1-\eta)}{2\sqrt{\mathrm{Fo}}}\right] + \mathrm{erfc}\left[\frac{(2n-1) + (1-\eta)}{2\sqrt{\mathrm{Fo}}}\right] \right\}.$$

We will consider the solution of problem (1)–(4) for a specimen of material on a metal backing, which is considered an ideal conductor (see Fig. 1c) [4–6]:

$$\theta(\eta, \mathrm{Fo}, \eta_0) = \frac{T(y,t) - T_0}{T_\mathrm{w} - T_0} = 1 - \sum_{n=1}^{\infty} A_n \sin(\mu_n \eta) \exp(-\mu_n^2 \mathrm{Fo})\,, \tag{8}$$

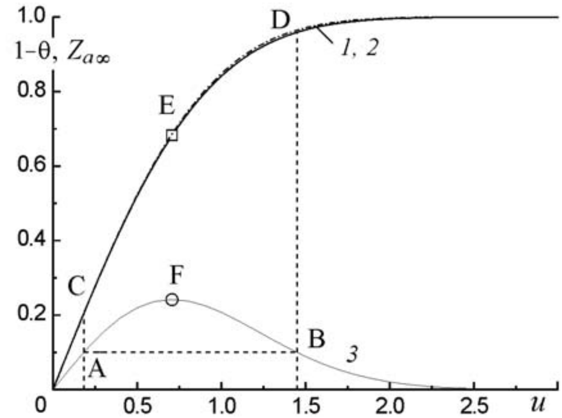


Fig. 2. Temperature θ and the coefficient of sensitivity $Z_{a\infty}$ as functions of u (semi-infinite body): 1, 2) (1–θ), F_{app}; 3) $Z_{a\infty}$. F — $Z_{a\infty}(u_{\max})$, E — $\theta(u_{\max})$.

$$\mu_n \tan \mu_n = \eta_0 \; ; \quad A_n = \frac{2\,(\mu_n^2 + \eta_0^2)}{\mu_n\,(\mu_n^2 + \eta_0^2 + \eta_0)} \; ; \quad \eta_0 = \frac{\rho c h}{\rho' c' h'} \, . \tag{9}$$

Solution (8) depends parametrically on the coefficient η_0, which is considered known.

Despite the complex structure, expressions (7) and (8) are exact, rather than approximate, for any values of (η, Fo). Consequently, direct use of Eqs. (7), (8) in solving an inverse heat conduction problem does not require the isolation of the heating stage (initial, transient, regular) and of the range of Fourier numbers at which these expressions are simplified to explicit ones to determine the thermophysical characteristics.

Analysis of the Coefficients of the Sensitivity of Solutions. From the viewpoint of planning and processing the results of a thermophysical experiment, it is desirable to know the optimum time at which experimental data contain a maximum of information on the sought-for parameters, and when the influence of the error of measurements on the determination of them is minimum. We will consider the dimensionless coefficients $Z_{a\infty}$, Z_{ah}, $Z_{ahh'}$ of the sensitivity of solutions (6)–(8) toward a change in the sought parameter a [7, 8]. Having taken the corresponding derivative with respect to it, we obtain

$$Z_{a\infty}(u) = \frac{a}{T_{\text{w}} - T_0} \frac{\partial T}{\partial a} = \frac{1}{\sqrt{\pi}}\, u \exp(-u^2) \, , \tag{10}$$

$$Z_{ah}(\eta, \text{Fo}) = \frac{a}{T_{\text{w}} - T_0} \frac{\partial T}{\partial a} = \sum_{n=1}^{\infty} A_n \cos[\mu_n (1 - \eta)]\,(\mu_n^2 \text{Fo}) \exp(-\mu_n^2 \text{Fo}) \, , \tag{11}$$

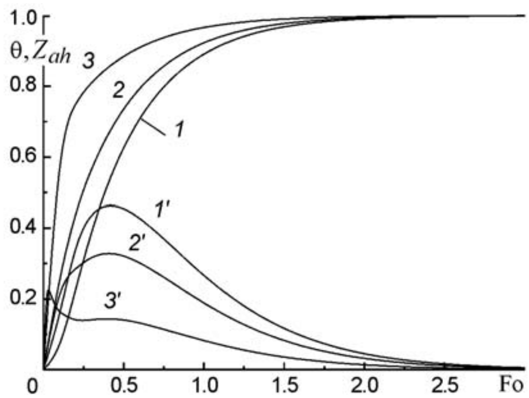
$$Z_{ahh'}(\eta, \text{Fo}, \eta_0) = \frac{a}{T_{\text{w}} - T_0} \frac{\partial T}{\partial a} = \sum_{n=1}^{\infty} A_n \sin(\mu_n \eta)\,(\mu_n^2 \text{Fo}) \exp(-\mu_n^2 \text{Fo}) \, . \tag{12}$$

The graphs of the temperature θ and sensitivity coefficients $Z_{a\infty}$, Z_{ah}, $Z_{ahh'}$ depending on the arguments u and Fo are presented in Figs. 2–4. Their analysis shows that the function $Z_{a\infty}(u)$ for a semi-infinite body has a nonmonotonic character with the presence of a maximum (the point F on curve 3, Fig. 2). Its coordinate can be found from the equation

$$\frac{\partial Z_{a\infty}}{\partial u} = \frac{1}{\sqrt{\pi}} \exp(-u^2)\,(1 - 2u^2) = 0 \, ,$$

TABLE 1. Coefficient of Sensitivity Z_a for a Single-Layer and Two-Layer Plates

η	Plate			Plate with backing, $\eta_0 = 1$		
	Fo_{max}	$\theta(Fo_{max})$	$Z_{ah}(Fo_{max})$	Fo_{max}	$\theta(Fo_{max})$	$Z_{ahh'}(Fo_{max})$
1.0	0.4	0.526	0.468	1.35	0.588	0.412
0.5	0.4	0.664	0.332	1.35	0.773	0.226



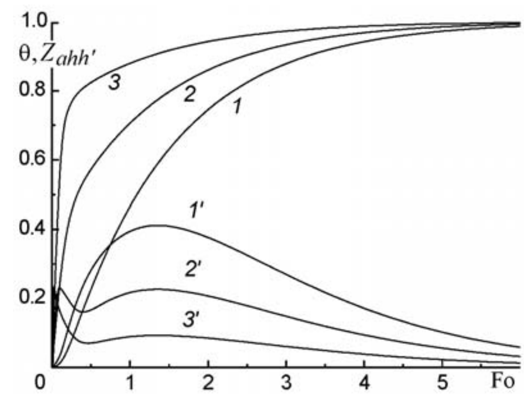


Fig. 3. Temperature θ and the coefficient of sensitivity Z_{ah} as functions of the Fo number (a plate of height h): 1–3) θ; 1′–3′) Z_{ah} at $\eta = 1$, 0.5, and 0.2, respectively.

Fig. 4. Temperature θ and the coefficient of sensitivity $Z_{ahh'}$ as functions of the Fo number (a plate with a layer of an ideal conductor, $\eta_0 = 1$): 1–3, θ; 1′–3′) $Z_{ahh'}$ at $\eta = 1$, 0.5, and 0.2, respectively.

from which it follows that $u_{max} = 1/\sqrt{2} \approx 0.707$ and $Z_{a\infty}(u_{max}) = \exp(-1/2)/\sqrt{2\pi} \approx 0.242$. The corresponding value of temperature is $\theta(u_{max}) = 1 - \text{erf}(1/\sqrt{2}) \approx 0.317$ (the point E on curve 1 in Fig. 2). Experimental information for determining the coefficient a is of greatest interest in the vicinity of the maximum of the function $Z_{a\infty}(u)$ (the points F and E in Fig. 2), since here the influence of the error of temperature measurements is minimum in comparison with other points. For example, assuming that $Z_{a\infty} \approx 0.1$, we obtain that the points of measurements of time lie in the range $0.18 < u < 0.45$ (points A and B), and of temperature θ, in the range $0.04 < \theta < 0.8$ (points C and D). Outside this range of the temperature curve, the sensitivity of the solution of the inverse heat conduction problem to measurement errors becomes stronger because of the weaker response of the solution (small values of $Z_{a\infty}$) to the change in the unknown parameter a.

For single- and two-layer (with a layer of an ideal conductor) plates in the behavior of the Z_{ah} and $Z_{ahh'}$ curves is also nonmonotonic, with a clearly expressed maximum (curves 1′ and 2′ in Figs. 3 and 4). For the plate with the backing the coefficient η_0 was assumed equal to unity, which by heat capacity corresponds to the same contribution from the specimen and the backing. Higher values of $\eta_0 > 10$ are typical of a very thin backing, and in this case the graphs in Fig. 4 approach the behavior of the curves in Fig. 3 (with no backing). The coordinates of the maximum of the curves and the corresponding values of $Z_{ah}(Fo_{max})$, $\theta(Fo_{max})$ for the rear side ($\eta = 1$) and the middle ($\eta = 0.5$) of the plate are given in Table 1. This provides information on the time and points on the temperature curve that carry a maximum of information on the thermal diffusivity coefficient a. Since the maxima of the Z_{ah} and $Z_{ahh'}$ curves are higher than of the $Z_{a\infty}$ curve, from this it follows that at the same level of the error of measurements the determination of a for a plate of finite thickness will be more accurate than for a semi-infinite body. A comparison of curves 1′–3′ in Figs. 3 and 4 also shows that for this purpose the rear side of the plate ($\eta = 1$) carries more information than the layers near the heated surface, since for this side the corresponding sensitivity coefficient is higher.

Technique of Determining Thermophysical Characteristics. In solving the inverse heat conduction problem within the framework of the model of a semi-infinite body it is useful to have the approximation of the function erf(u) which would admit an explicit expression of the argument u. We will consider the approximation [11] that graphically coincides with exact values (see Fig. 2, curves 1 and 2):

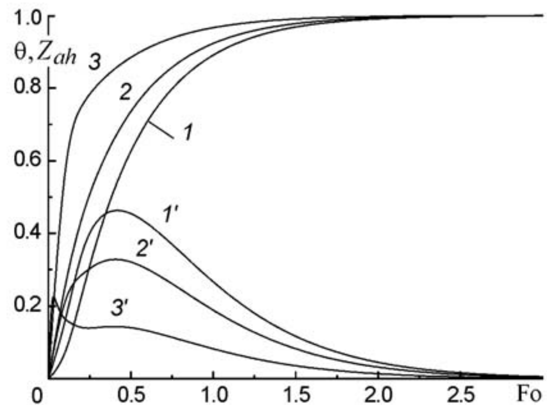


Fig. 5. Error of approximation $F(u)$ = erf(u) (semi-infinite body): 1) ΔF (absolute); 2) δF (relative).

$$F(u) = \operatorname{erf}(u)\,, \quad F_{\text{app}}(u) = \sqrt{1 - \exp\{-4u^2/\pi\}}\,. \tag{13}$$

Its absolute, $\Delta F = F - F_{\text{app}}$, and relative, $\delta F = \Delta F/F$, errors are shown in Fig. 5 and lie in the ranges $-0.006 < \Delta F < 0$, $-0.007 < \delta F < 0$, which is quite appropriate for practical application. The use of (13) makes it possible to express explicitly the sought-for coefficient a, and for N measurements in time for a single sensor to obtain the following formula:

$$a = \frac{y_j^2}{4N} \sum_{i=1}^{N} \frac{1}{t_i c_i}\,, \quad c_i = -\frac{\pi}{4} \ln\left[1 - (1 - Y_i)^2\right], \tag{14}$$

where Y_i represents experimental values of temperature at time t_i.

For M sensors with the coordinates y_j ($1 \le j \le M$), Eq. (14) will be written as

$$a = \left\{ \sum_{j=1}^{M} \sum_{i=1}^{N_j} \frac{y_j^2}{t_{i,j} c_{i,j}} \right\} \Big/ \left\{ 4 \sum_{j=1}^{M} N_j \right\}. \tag{15}$$

We will consider the general case of determining the coefficient a from the experimental values of temperature $T_{\text{exp}}(y_j,\ t_{i,j})$ at time moments $t_{i,j}$ for M sensors with the coordinates y_j ($1 \le j \le M$), For this purpose we calculate

$$Y(y_j, t_{i,j}) = \frac{T_{\text{exp}}(y_j, t_{i,j}) - T_0}{T_{\text{w}} - T_0}\,.$$

We will compose the functional $J(a)$ (5) by the least squares method. Using the condition of the minimum of the functional [8, 12], we obtain a nonlinear transcendental equation to determine the sought-for coefficient a:

$$\frac{\partial J}{\partial a} = -2 \sum_{j=1}^{M} \sum_{i=1}^{N_j} \left[Y(y_j, t_{i,j}) - \theta(y_j, t_{i,j})\right] \left(\frac{\partial \theta}{\partial a}\right)_{i,j} = 0\,. \tag{16}$$

The cofactor in parentheses in Eq. (16) has the sense of the above-considered coefficient of sensitivity at the corresponding points in time. Equation (16) is solved numerically.

Examples of Application of the Technique. To verify the technique, we will consider the following organization of a thermophysical experiment. Let the material studied have the thermophysical characteristics ρ = 100 kg/m^3, λ = 0.1 W/(m·K), and c = 10^3 J/(kg·K), with the thermal diffusivity coefficient being equal to $a = 1 \cdot 10^{-6}$

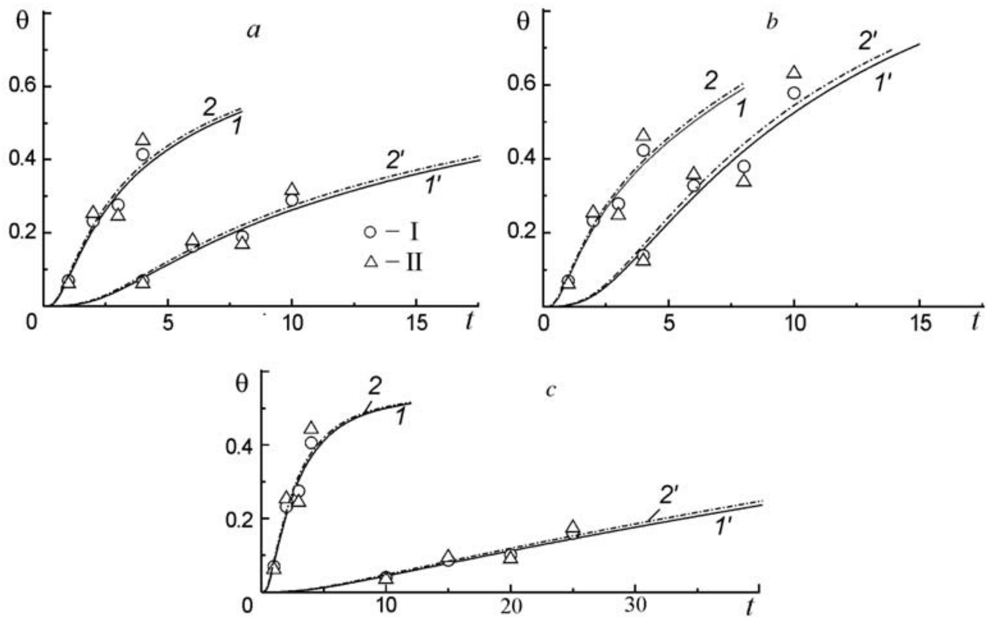


Fig. 6. Temperature at y_1 = 2.5 mm and y_2 = 5 mm as a function of time t [a) semi-infinite body; b) plate of thickness h = 5 mm; c) plate on metal backing, h = 5 mm, h' = 1 mm]: I, II, experimental data with error ε = 0.1, 0.2; 1, 1′, exact data; 2, 2′, recovered values (at ε = 0.2).

TABLE 2. Recovered Values of the Coefficient a. Semi-Infinite Body

ε	$a \cdot 10^6$	$a \cdot 10^6$, Eq. (15)	$a \cdot 10^6$	$a \cdot 10^6$, Eq. (15)
	four points		three points	
0	1.0	1.025	1.0	1.029
0.1	1.024	1.044	0.973	1.005
0.2	1.047	1.071	0.946	0.985

TABLE 3. Recovered Values of the Coefficient a. Single-Layer and Two-Layer Plates

ε	Plate, $a \cdot 10^6$		Plate with backing, η_0 = 0.2, $a \cdot 10^6$	
	four points	three points	four points	three points
0	1.0	1.0	1.0	1.0
0.1	1.022	0.974	1.025	0.972
0.2	1.043	0.947	1.049	0.945

m^2/sec. The plate thickness is h = 5 mm, and the thickness of the metal backing is h' = 1 mm. The thermophysical characteristics of the backing are: ρ' = 2500 kg/m^3, c' = 10^3 J/(kg·K); at these values the coefficient η_0 = 0.2.

Let the temperature sensors be located at a depth of y_1 = 2.5 mm and y_2 = 5 mm. An exact solution for the model of a semi-infinite body is shown by solid curve 1 in Fig. 6a. As "experimental" data (symbols) for the solution of the inverse heat conduction problem here and below we took the perturbed values of the exact solution $Y_{j,i} = \theta_{j,i}(1 \pm \varepsilon)$ with the relative error ε at several points in time. For the first sensor (y_1 = 2.5 mm) the arbitrarily selected values of time were $t_{i,1}$ = (1; 2; 3; 4) sec, and for the second (y_2 = 5 m), $t_{i,2}$ = (4; 6; 8; 10) sec. Moreover, to assign a nonsymmetric error in data we also used the set of the first three points in time for each sensor. The results of the solution of the inverse heat conduction problem on the basis of Eq. (16) and approximate formula (15) are presented in Table 2. It is seen that in the absence of error in the initial data (ε = 0) the coefficient a is recovered exactly. In the presence of a considerable error in the data of 20% (ε = 0.2), the error of the solution of the inverse heat conduction problem does not exceed 4–5%. This is confirmed by the behavior of curves 2, 2′ (Fig. 6a) obtained with the use of the recovered value of the coefficient a (at $\varepsilon = 0.2$). It should be noted that usually the real error of measurements in experiments does not exceed 5% ($\varepsilon < 0.05$). The calculation of a by approximate equation (15) also provides an accuracy is adequate for practice.

For the single-layer plate the same set of time points was used; with respect to the Fourier number these are Fo = 0.04–0.16 and Fo = 0.16–0.4 for the first and second sensors. Their location corresponds to the middle and rear side of the plate. An analysis of Fig. 6b and Table 3 shows that the accuracy of the recovery of the coefficient a on the basis of series (7) is very high. For the symmetric error in the initial data of 20% (four points) it constitutes ~4%, and for the nonsymmetric one (three points), ~5%.

The presence of a metal backing increases the duration of an experiment. Therefore for the rear surface of the plate another set of points in time was used as experimental ones $t_{i,2}$ = (10; 15; 20; 25) sec; the former set remained for the first sensor. The results of the solution of the inverse heat conduction problem with the use of the solution in the form of series (8) are presented for this case in Table 3 and shown in Fig. 6c. It is seen that the 20% perturbation of the exact solution gave an error in the determination of the coefficient a of about 5% at four points and of about 6% at three points. The indicated level of the error in the determination of the coefficient a for the above-considered three types of specimens is also maintained when one sensor is used. An increase in the number of time points and sensors, other conditions being equal, makes it possible to raise the correctness of determining the thermal diffusivity coefficient $a(\lambda)$.

The proposed technique of solving the inverse coefficient problem was used in processing the data of thermal experiments dealing with the investigation and monitoring of the thermophysical characteristics of foamed cokes of heat- and fire proof coatings SGK-1 and SGK-2 [13, 14].

CONCLUSIONS

1. From a single methodological viewpoint, an inverse problem of determining the thermal diffusivity (thermal conductivity) coefficient of a material by measuring its temperature at its depth has been solved.

2. The range of the times of measurements, when experimental data are most informative as concerns the thermal diffusivity coefficient and the influence of the error in its determination is minimum, has been determined.

3. The proposed technique of processing the data of a thermophysical experiment does not require the smoothing of experimental data and the selection of a stage on the thermal process and allows one to expand the range of Fourier-number-related measurements and increase the accuracy of the solution of the inverse coefficient problem.

NOTATION

a, thermal diffusivity coefficient, m^2/sec; A_n, amplitude coefficient; c, specific heat, J/(kg·K); $F(u)$, $F_{app}(u)$, function erf(u) and its approximation; Fo, Fourier number; h, h', thickness of the plate and backing, m; $J(a)$, functional of the residual dependent on a; M, number of sensors; N_j, number of processed points in time for the j sensor; t, time, sec; T, temperature, K; u, self-similar Boltzmann variable; y, coordinate directed inside the material, m; $Y_{j,i}$, experimental values of temperature for the j sensor at the time instant i; $Z_{a\infty}$, Z_{ah}, $Z_{ahh'}$, coefficient of the sensitivity of the solution to the change in the coefficient a for a semi-infinite body, plane plate, a plate with a backing; ε, error of "experimental" data; η, dimensionless coordinate; η_0, ratio of heat capacities for the plate and backing; θ, dimensionless temperature; λ, thermal conductivity, W/(m·K); μ_n, roots of the transcendental equation, characteristic numbers; ξ, integration variable; ρ, density of material, kg/m^3. Subscripts: 0, initial conditions; ′ , backing; app, approximation; i, points in time for the j sensor of temperature; max, maximum; n, ordinal number of the term in the series; w, wall.

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